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General (anti-)de Sitter black holes in five dimensions

Dietmar Klemm
Dipartimento di Fisica dell'Università di Milano and
INFN, Sezione di Milano
Via Celoria 16, 20133 Milano, Italy
E-mail: dietmar.klemm@mi.infn.it

Wafic A. Sabra
Center for Advanced Mathematical Sciences (CAMS) and
Physics Department, American University of Beirut, Lebanon
E-mail: ws00@aub.edu.lb

Abstract: We find a general class of rotating charged black hole solutions to $N = 2$, $D = 5$ gauged supergravity coupled to vector supermultiplets. The supersymmetry properties of these solutions are studied, and their mass and angular momenta are obtained. We also compute the stress tensor of the dual $D = 4$, $\mathcal{N} = 4$ super Yang-Mills theory in the limit of strong 't Hooft coupling. It is shown that closed timelike curves occur outside the horizon, indicating loss of unitarity in the dual CFT. For imaginary coupling constant of the gravitini to the gauge fields, one can obtain multi-centered rotating charged de Sitter black holes. Some physical properties of these solutions are also discussed.

Keywords: AdS-CFT Correspondance, Black Holes, Black Holes in String Theory, Supergravity Models.

Contents

1. Introduction

Since the advent of the duality between string theory (or, for energies much smaller than the string scale, supergravity) on anti-de Sitter (AdS) spaces and supersymmetric conformal field theories residing on the conformal boundary of AdS [1], there has been a renewed interest in the study of AdS black holes. The reason for this is mainly based on the possibility to study the nonperturbative structure of these field theories by means of classical supergravity solutions. One hope is that this correspondence will eventually lead to a deeper insight into confinement in QCD, or at least into the strong coupling regime of gauge theories with fewer supersymmetries. A standard example for the exploration of the nonperturbative regime of conformal field theories using supergravity solutions is the Hawking-Page phase transition [2], which can be interpreted as a thermal phase transition from a confining to a deconfining phase in the dual $D = 4$, $\mathcal{N} = 4$ super Yang-Mills theory [3]. Up to now, the study of AdS black holes in the context of the AdS/CFT correspondence has been extended in various directions [4]–[10], and several interesting observations were made, like the striking resemblance of the phase structure of Reissner-Nordström-AdS black holes to that of a van der Waals-Maxwell liquid-gas system [4], the possible appearance of so-called "precursor" states [11] in the CFT duals of hyperbolic black holes [5], or the disappearance of the Hawking-Page transition for finite ‘t Hooft coupling [6]. Besides, by considering Kerr-AdS black holes, the authors of [8] found an explicit example of a field theory which is scale invariant but not conformally invariant. Finally, it is worth mentioning that recently there has been an interest into black strings in AdS space [12, 13], because these describe a flow of four-dimensional $\mathcal{N} = 4$ SYM theory on $\mathbb{R}^2 \times \Sigma_g$ (where Σ_g denotes a genus g Riemann surface) to a two-dimensional CFT in the infrared [13, 14].

These results motivate a further research in this field, in particular it is of interest to find new solutions to various gauged supergravity theories. Black holes which preserve some of the original supersymmetries could correspond to an expansion around nonzero vacuum expectation values of certain CFT operators. In this paper we will be mainly concerned in studying charged rotating solutions to the theory of $N = 2$, $D = 5$ gauged supergravity coupled to an arbitrary number of abelian vector multiplets. Such solutions were hitherto unknown.

This paper is organized as follows: in section 2 we briefly review $N = 2$, $D = 5$ gauged supergravity theories coupled to vector multiplets. In section 3 we derive general solutions for these theories which admit Killing spinors. In section 4 black holes with rotational symmetry in two orthogonal planes are explicitly constructed, and in section 5 we discuss the physics of the obtained solutions, i. e. we compute their mass and angular momenta, and show that there exist closed timelike curves in the region outside the horizon. The possible relationship of this causality violation with loss of unitarity in the dual conformal field theory is also discussed. Furthermore, the stress tensor of this CFT is determined. Finally, section 6 is devoted to the de Sitter counterparts obtained by analytically continuing to imaginary coupling constant of the gravitini. In this case, a Smarr formula for the black holes is derived. We conclude with some final remarks.

2. $N = 2$, $D = 5$ gauged supergravity

The theory we shall deal with is $N = 2$, $D = 5$ gauged supergravity coupled to an arbitrary number n of abelian vector supermultiplets. The bosonic part of the lagrangian is given by

$$e^{-1}\mathcal{L} = \frac{1}{2}R + g^2 V - \frac{1}{4}G_{IJ}F^I_{\mu\nu}F^{J\mu\nu} - \frac{1}{2}\mathcal{G}_{ij}\partial_\mu\phi^i\partial^\mu\phi^j + \frac{e^{-1}}{48}\epsilon^{\mu\nu\rho\sigma\lambda}C_{IJK}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda\,, \tag{2.1}$$

where μ, ν are spacetime indices, R is the scalar curvature, $F^I_{\mu\nu}$ denote the abelian field-strength tensors, and e is the determinant of the Fünfbein e^a_μ. The scalar potential V is given by

$$V(X) = V_I V_J \left(6X^I X^J - \frac{9}{2}\mathcal{G}^{ij}\partial_i X^I \partial_j X^J\right)\,. \tag{2.2}$$

The X^I are functions of the real scalar fields subject to the condition $\mathcal{V} = 1/6 C_{IJK} \times X^I X^J X^K = 1$. The gauge and the scalar couplings are determined by the homogeneous cubic polynomial $\mathcal{V}$ which defines a "very special geometry". They are given by

$$\begin{aligned} G_{IJ} &= -\frac{1}{2}\partial_I \partial_J \log \mathcal{V}\Big|_{\mathcal{V}=1}, \\ \mathcal{G}_{ij} &= \partial_i X^I \partial_j X^J G_{IJ}\Big|_{\mathcal{V}=1}, \end{aligned} \tag{2.3}$$

where ∂_i and ∂_I refer, respectively, to partial derivatives with respect to the scalar fields ϕ^i and $X^I = X^I(\phi^i)$.

Furthermore, the constants V_I that arise in (2.2) specify the appropriate linear combination of the vectors that comprise the $N = 2$ graviphoton, $\mathcal{A}_\mu = V_I A^I_\mu$.

For Calabi-Yau compactifications of M-theory, $\mathcal{V}$ denotes the intersection form, and X^I and $X_I \equiv 1/6 C_{IJK} X^J X^K$ correspond to the size of the two- and four-cycles of the Calabi-Yau threefold respectively. Here C_{IJK} are the intersection numbers of the threefold. In the Calabi-Yau cases, n is given by the Hodge number $h_{(1,1)}$. Some useful relations from very special geometry are

$$\begin{aligned} X^I X_I &= 1, & \partial_i X_I &= -\frac{2}{3} G_{IJ} \partial_i X^J, \\ X_I &= \frac{2}{3} G_{IJ} X^J, & X_I \partial_i X^I &= X^I \partial_i X_I = 0. \end{aligned} \tag{2.4}$$

In gauged supergravity theories, the supersymmetry transformations of the gravitino and gauginos in a bosonic background are given by [15]

$$\begin{aligned} \delta\psi_\mu &= \left[\mathcal{D}_\mu + \frac{i}{8} X_I (\Gamma_\mu{}^{\nu\rho} - 4\delta^\nu_\mu \Gamma^\rho) F^I_{\nu\rho} + \frac{1}{2} g \Gamma_\mu X^I V_I\right]\epsilon, \\ \delta\lambda_i &= \left[-\frac{1}{4} G_{IJ} \Gamma^{\mu\nu} F^J_{\mu\nu} + \frac{3i}{4}\Gamma^\mu \partial_\mu X_I + \frac{3i}{2} g V_I\right] \partial_i X^I \epsilon. \end{aligned} \tag{2.5}$$

Here $\mathcal{D}_\mu$ is the fully gauge and gravitationally covariant derivative,[1]

$$\mathcal{D}_\mu \epsilon = \left[\nabla_\mu - \frac{3i}{2} g V_I A^I_\mu\right]\epsilon. \tag{2.6}$$

The ungauged theory is obtained in the (smooth) limit $g \to 0$.

3. General solutions admitting killing spinors

The Einstein equations with positive cosmological constant admit de Sitter space as vacuum solution, with a metric that can be written in the form

$$ds^2 = -(1 - g^2 r^2) dt^2 + \frac{dr^2}{1 - g^2 r^2} + r^2 d\Omega_3^2, \tag{3.1}$$

[1] We use the metric $\eta^{ab} = (-,+,+,+,+)$, $\{\Gamma^a, \Gamma^b\} = 2\eta^{ab}$, $\nabla_\mu = \partial_\mu + \frac{1}{4}\omega_{\mu ab}\Gamma^{ab}$, $\omega_{\mu ab}$ is the spin connection, and $\Gamma^{a_1 a_2 \cdots a_n} = \Gamma^{[a_1}\Gamma^{a_2}\cdots\Gamma^{a_n]}$, where antisymmetrization is taken with unit weight.

where $d\Omega_3^2$ is the standard metric on the unit three-sphere. Denoting by $d\vec{x}^2$ the flat line element in four dimensions, (3.1) can be rewritten in the cosmological form

$$ds^2 = -dt^2 + e^{2gt} d\vec{x}^2 \,, \tag{3.2}$$

which is related to the above metric by changing the coordinates according to

$$r' = re^{gt} \,, \quad dt = dt' - \frac{gr'}{1 - g^2 r'^2} dr' \,, \tag{3.3}$$

and then dropping the primes. Multi-centered black hole solutions asymptotic to (3.2) were found in [16, 17]. These solutions are given by

$$\begin{aligned}
ds^2 &= -e^{-4U} dt^2 + e^{2U} e^{2gt} d\vec{x}^2 \,, && (3.4)\\
A_t^I &= e^{-2U} X^I \,, && (3.5)\\
X_I &= \frac{1}{3} e^{-2U} H_I \,, && (3.6)\\
H_I &= 3V_I + \sum_{i=1}^{N} \frac{q_{Ii}}{|\vec{x}_i - \vec{x}|^2} e^{-2gt} \,, && (3.7)
\end{aligned}$$

where the q_{Ii} are related to the electric charges. These solutions resemble those of the ungauged theory [18] but with the spatial coordinates modified by an exponential time-dependent factor.

Under the change of variables

$$r' = re^{gt}, \qquad dt = dt' - \frac{gr' e^{6U}}{1 - g^2 r'^2 e^{6U}} dr' \,, \tag{3.8}$$

we get (after dropping the primes) for the metric

$$ds^2 = -e^{-4U}(1 - g^2 r^2 e^{6U}) dt^2 + \frac{e^{2U} dr^2}{1 - g^2 r^2 e^{6U}} + e^{2U} r^2 d\Omega_3^2 \,, \tag{3.9}$$

which is the de Sitter version of the solutions found in [15].

From the preceding analysis it is clear that it might be advantageous to analyze more general charged black hole solutions in the time-dependent cosmological frame. Therefore, our starting point is the ansatz

$$ds^2 = -e^{-4U}(dt + G(t) w_m dx^m)^2 + e^{2U} F^2(t) d\vec{x}^2 \,, \tag{3.10}$$

where $U = U(x,t)$, $w_m = w_m(x)$, and G, F are time-dependent functions. The Fünfbein and its inverse for the metric (3.10) can be taken as

$$\begin{aligned}
{e_t}^0 &= e^{-2U} \,, & e_m^{\ 0} &= G e^{-2U} w_m \,, \\
{e_t}^a &= 0 \,, & e_m^{\ a} &= e^U F \delta_m^a \,, \\
{e^t}_0 &= e^{2U} \,, & {e^t}_a &= -\frac{G}{F} e^{-U} w_m \delta_a^m \,, \\
e^m_{\ 0} &= 0 \,, & e^m_{\ a} &= \frac{1}{F} e^{-U} \delta_a^m \,.
\end{aligned} \tag{3.11}$$

For the spin connection one obtains

$$
\begin{aligned}
\omega_t^{0a} &= \frac{e^{-3U}}{F}\delta^{am}[(-\dot{G}+2G\dot{U})w_m - 2\partial_m U]\,, \\
\omega_n^{0a} &= e^{3U}(\dot{F}+F\dot{U})\delta_n^a - \frac{G}{2F}e^{-3U}\delta^{am}(\partial_n w_m - \partial_m w_n) + \frac{e^{-3U}G}{F}\delta^{am}w_n \times \\
&\quad \times[(-\dot{G}+2G\dot{U})w_m - 2\partial_m U], \\
\omega_t^{ab} &= -\frac{G}{2F^2}e^{-6U}\delta^{nb}\delta^{ma}(\partial_n w_m - \partial_m w_n)\,, \\
\omega_n^{ab} &= (\delta^{mb}\delta_n^a - \delta^{ma}\delta_n^b)\left[\partial_m U - w_m G\left(\dot{U}+\frac{\dot{F}}{F}\right)\right] - \frac{G^2}{2F^2}e^{-6U}w_n\delta^{pb}\delta^{ma} \times \\
&\quad \times(\partial_p w_m - \partial_m w_p)\,.
\end{aligned} \tag{3.12}
$$

We will look for configurations which admit Killing spinors. The Killing spinor equations are obtained by setting to zero the supersymmetry transformations of the Fermi fields (in a bosonic background) for the choice $\Gamma_0\epsilon = i\epsilon$. From the time component of the gravitino supersymmetry transformation, one thus obtains

$$
\begin{aligned}
\delta\psi_t &= \partial_t\epsilon - \frac{i}{2}\omega_t^{0a}\Gamma_a\epsilon - \frac{1}{4F^2}e^{-6U}\delta_a^m\delta_b^n(G(\partial_n w_m - \partial_m w_n) + e^{2U}X_I F_{mn}^I)\Gamma^{ab}\epsilon - \\
&\quad - \frac{i}{2F}e^{-U}X_I F_{tm}^I\delta_a^m\Gamma^a\epsilon + \frac{i}{2}ge^{-2U}X^I V_I\epsilon - \frac{3i}{2}gV_I A_t^I\epsilon.
\end{aligned} \tag{3.13}
$$

Using the relation

$$
\Gamma^{ab} = -\frac{i}{2}\epsilon^{abcd0}\Gamma_{cd}\Gamma_0\,, \tag{3.14}
$$

we then get from the vanishing of (3.13) the equations

$$
\begin{aligned}
(\partial_t + \frac{i}{2}ge^{-2U}X^I V_I - \frac{3i}{2}gV_I A_t^I)\epsilon &= 0\,, \\
-e^{-2U}[(-\dot{G}+2G\dot{U})w_m - 2\partial_m U] - X_I F_{tm}^I &= 0\,, \\
(X_I F_{mn}^I)^- &= (\partial_n Q_m - \partial_m Q_n)^-\,,
\end{aligned} \tag{3.15}
$$

where $Q_n \equiv Ge^{-2U}w_n$ and $F_{mn}^- = F_{mn} - {}^\star F_{mn}$. Note that the second equation in (3.15) can be rewritten as

$$
X_I F_{tm}^I = \partial_t Q_m - \partial_m e^{-2U}\,. \tag{3.16}
$$

Thus, using also the relations $X_I\partial X^I = 0$, $X_I X^I = 1$, we deduce the following solution for the gauge fields,

$$
A_m^I = e^{-2U}w_m X^I, \qquad A_t^I = e^{-2U}X^I\,, \tag{3.17}
$$

and therefore the first and the third relation in (3.15) reduce to

$$
(\partial_t - ige^{-2U}V_I X^I)\epsilon = 0\,, \tag{3.18}
$$

$$
X_I F_{mn}^I = (\partial_n Q_m - \partial_m Q_n)\,. \tag{3.19}
$$

JHEP02(2001)031

We turn now to the spatial component of the gravitino supersymmetry transformation, which for our ansatz reads

$$\delta\psi_m = \partial_m\epsilon + \frac{1}{4}(\omega_m^{ab}\Gamma_{ab} + 2i\omega_m^{a0}\Gamma_a)\epsilon + \frac{i}{8}X_I(\Gamma_m^{\ np}F^I_{np} + 2\Gamma_m^{\ nt}F^I_{nt})\epsilon - \\ -\frac{i}{2}X_I(\Gamma^n F^I_{mn} + \Gamma^t F^I_{mt})\epsilon + \frac{1}{2}g\Gamma_m X^I V_I\epsilon - \frac{3i}{2}gV_I A^I_m\epsilon\,. \tag{3.20}$$

By substituting the relations of the gauge fields, we find that the Γ_a and Γ_{ab} terms vanish provided

$$(\partial_m w_n - \partial_n w_m)^- = 0\,, \tag{3.21}$$

$$-ie^{2U}(\dot{F} + F\dot{U}) + gX^I V_I F = 0\,, \tag{3.22}$$

$$GF^2 = 1\,. \tag{3.23}$$

Furthermore, we obtain the differential equation

$$(\partial_m + \partial_m U)\,\epsilon = 0\,. \tag{3.24}$$

Using (3.22), equation (3.18) becomes

$$(\partial_t + \frac{\dot{F}}{F} + \dot{U})\epsilon = 0\,. \tag{3.25}$$

Therefore the configuration found admits the Killing spinor

$$\epsilon = \frac{e^{-U}}{F}\epsilon_0\,, \tag{3.26}$$

where ϵ_0 is a constant spinor satisfying the constraint $\Gamma_0\epsilon_0 = i\epsilon_0$.

In order to fix the scalar fields and therefore the gauge fields in terms of space-time functions, we turn to the gauge field equations of motion. These are given by

$$\partial_\nu(eG_{IJ}g^{\mu\rho}g^{\nu\sigma}F^J_{\rho\sigma}) = \frac{1}{16}C_{IJK}\epsilon^{\mu\nu\rho\sigma\kappa}F^J_{\nu\rho}F^K_{\sigma\kappa}\,. \tag{3.27}$$

After some lengthy calculations one finds

$$e^{2U}X_I = \frac{1}{3}H_I\,, \tag{3.28}$$

with the harmonic functions $H_I = h_I + \frac{q_I}{r^2}G$.

Finally one can fix h_I and G by turning to the equation obtained from the vanishing of the supersymmetry variation for the gauginos. This yields

$$\left(-G_{IJ}\Gamma^{\mu\nu}F^J_{\mu\nu} + 3i\Gamma^\mu\partial_\mu X_I + 6igV_I\right)\partial_i X^I\epsilon = 0\,. \tag{3.29}$$

Requiring the term independent of the Γ^a to be zero gives

$$(2igV_I + e^{2U}\partial_t X_I)\partial_i X^I\epsilon = 0\,. \tag{3.30}$$

The vanishing of the Γ^{ab} and Γ^a terms yields

$$\left(G_{IJ}\partial_i X^I F^J_{mn} + \frac{3G}{2}e^{-2U}(\partial_m X_I w_n - \partial_n X_I w_m)\partial_i X^I\right)^- = 0\,,$$
$$\left(-\frac{1}{F}G_{IJ}e^U F^J_{tm} - \frac{3G}{2F}e^{-U}w_m\partial_t X_I + \frac{3}{2F}e^{-U}\partial_m X_I\right)\partial_i X^I\epsilon = 0\,.$$

The above equations are satisfied for our ansatz. Relation (3.30) fixes $G(t) = e^{2igt}$ and $h_I = 3V_I$. If we define the rescaled coordinates

$$Y_I = e^{2U}X_I, \qquad Y^I = e^U X^I\,, \tag{3.31}$$

then the underlying very special geometry implies that

$$Y_I Y^I = e^{3U}X_I X^I = e^{3U} = \mathcal{V}(Y) = \frac{1}{3}C_{IJK}Y^I Y^J Y^K\,. \tag{3.32}$$

To summarize, we have found a black hole solution which breaks half of supersymmetry, given by

$$\begin{aligned}
ds^2 &= -\mathcal{V}^{-4/3}(Y)(dt + e^{2igt}w_m dx^m)^2 + \mathcal{V}^{2/3}(Y)e^{-2igt}d\vec{x}^2\,,\\
\frac{1}{2}C_{IJK}Y^J Y^K &= H_I = h_I + \frac{q_I}{r^2}e^{2igt}\,,\\
A^I_t &= \frac{Y^I}{\mathcal{V}(Y)}\,,\\
A^I_m &= e^{2igt}w_m A^I_t\,,\\
(\partial_m w_n - \partial_n w_m)^- &= 0\,.
\end{aligned} \tag{3.33}$$

Clearly the solution depends very much on the choice of the model under consideration. Explicit solutions can be found for the case with no vector multiplets, the STU model and the models considered in [19]. The only additional feature here is the dependence of the fields on time. At first sight, the complex time-dependence occurring in (3.33) seems awkward. This is certainly no problem in de Sitter space, where the coupling constant g is taken to be imaginary. Then all fields are real, and one can also have multi-centered black holes.[2] However, for the one-centered de Sitter solution[3], as we shall see in the next section, one can go to a set of new coordinates in which the metric depends only on g^2, whereas the gauge fields and scalars are independent of g, and therefore the anti-de Sitter rotating solution can then be simply obtained by replacing g^2 with $-g^2$ [17], so that all fields are real.

[2]For the case without vector multiplets, these have been obtained in [20]. For further discussion see [17].

[3]Which is obtained from (3.33) by the analytical continuation $g \to ig$.

4. Solutions with rotational symmetry

Using the above general ansatz one can construct models with rotational symmetry. If we introduce spherical coordinates by

$$\begin{aligned} x^1 + ix^2 &= r\sin\theta e^{i\phi}\,, \\ x^3 + ix^4 &= r\cos\theta e^{i\psi}\,, \\ d\vec{x}^2 &= dr^2 + r^2 d\Omega^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\psi^2)\,, \end{aligned} \tag{4.1}$$

and specialize to solutions with rotational symmetry in two orthogonal planes, i.e.

$$w_\phi = w_\phi(r,\theta)\,, \qquad w_\psi = w_\psi(r,\theta)\,, \qquad w_r = w_\theta = 0\,, \tag{4.2}$$

then the self-duality condition of the field strength of w implies [21]

$$\begin{aligned} \partial_r w_\phi + \frac{\tan\theta}{r}\partial_\theta w_\psi &= 0\,, \\ \partial_\theta w_\phi - r\tan\theta\partial_r w_\psi &= 0\,. \end{aligned} \tag{4.3}$$

For an asymptotically decaying solution, one finds[4]

$$w_\phi = -\frac{\alpha}{r^2}\sin^2\theta\,, \qquad w_\psi = \frac{\alpha}{r^2}\cos^2\theta\,. \tag{4.4}$$

If we analytically continue to de Sitter space, $g \to ig$, the solution (3.33) reads

$$ds^2 = -e^{-4U}\left(dt - e^{-2gt}\frac{\alpha}{r^2}\sin^2\theta d\phi + e^{-2gt}\frac{\alpha}{r^2}\cos^2\theta d\psi\right)^2 + e^{2U}e^{2gt}(dr^2 + r^2 d\Omega^2)\,. \tag{4.5}$$

Introducing the new coordinates (r', t', ϕ', ψ') according to

$$\begin{aligned} r' &= re^{gt}\,, & dt &= dt' + f(r')dr'\,, \\ d\phi &= d\phi' + h(r')dr'\,, & d\psi &= d\psi' - h(r')dr'\,, \end{aligned} \tag{4.6}$$

where

$$\begin{aligned} f(r') &= \frac{-gr'\left(e^{6U} - \alpha^2/r'^6\right)}{1 - g^2r'^2e^{6U} + g^2\alpha^2/r'^4}, \\ h(r') &= \frac{\alpha g}{r'^3\left(1 - g^2r'^2e^{6U} + g^2\alpha^2/r'^4\right)}, \end{aligned}$$

we obtain (after dropping the primes) the metric in stationary form,

$$\begin{aligned} ds^2 &= e^{2U}g^2r^2dt^2 - e^{-4U}(dt - \frac{\alpha}{r^2}\sin^2\theta d\phi + \frac{\alpha}{r^2}\cos^2\theta d\psi)^2 + \\ &+ e^{2U}\left(\frac{dr^2}{1 - g^2r^2e^{6U} + g^2\alpha^2/r^4} + r^2d\Omega^2\right)\,. \end{aligned} \tag{4.7}$$

[4]This amounts in cartesian coordinates to $w_1 = \alpha x^2/r^4$, $w_2 = -\alpha x^1/r^4$, $w_3 = -\alpha x^4/r^4$, $w_4 = \alpha x^3/r^4$.

For the gauge fields in the new coordinates, one gets

$$A^I_\phi = -e^{-2U} X^I \frac{\alpha}{r^2} \sin^2\theta\,, \qquad A^I_\psi = e^{-2U} X^I \frac{\alpha}{r^2} \cos^2\theta\,, \qquad A^I_t = e^{-2U} X^I\,, \tag{4.8}$$

so they are now time-independent. As a special case, one obtains the solution of the five-dimensional Einstein-Maxwell theory considered in [20].

By replacing g^2 with $-g^2$, one can now obtain the rotating AdS black holes for all supergravity theories with vector multiplets, which is given by

$$\begin{aligned} ds^2 &= -e^{2U} g^2 r^2 dt^2 - e^{-4U}(dt - \frac{\alpha}{r^2}\sin^2\theta d\phi + \frac{\alpha}{r^2}\cos^2\theta d\psi)^2 + \\ &+ e^{2U}\left(\frac{dr^2}{1 + g^2 r^2 e^{6U} - \frac{g^2\alpha^2}{r^4}} + r^2 d\Omega^2\right)\,. \end{aligned} \tag{4.9}$$

As an example, consider the so-called $STU = 1$ model[5] ($X^0 = S, X^1 = T, X^2 = U$). Using $e^{2U} X_I = H_I/3$, we get the following equations for the moduli fields,

$$\begin{aligned} e^{2U} TU &= H_0 = h_0 + \frac{q_0}{r^2}, \\ e^{2U} SU &= H_1 = h_1 + \frac{q_1}{r^2}, \\ e^{2U} ST &= H_2 = h_2 + \frac{q_2}{r^2}. \end{aligned} \tag{4.10}$$

Equation (4.10) together with the fact that $STU = 1$ implies that the metric and the moduli fields are given by

$$e^{6U} = H_0 H_1 H_2\,, \tag{4.11}$$

and

$$S = \left(\frac{H_1 H_2}{H_0^2}\right)^{1/3}, \qquad T = \left(\frac{H_0 H_2}{H_1^2}\right)^{1/3}, \qquad U = \left(\frac{H_0 H_1}{H_2^2}\right)^{1/3}. \tag{4.12}$$

For anti-de Sitter supergravity, one obtains the charged rotating one-centered solution of the $STU = 1$ model,

$$\begin{aligned} ds^2 &= -(H_0H_1H_2)^{1/3} g^2 r^2 dt^2 - (H_0H_1H_2)^{-2/3}\left(dt - \frac{\alpha}{r^2}\sin^2\theta d\phi + \frac{\alpha}{r^2}\cos^2\theta d\psi\right)^2 + \\ &+ (H_0H_1H_2)^{1/3}\left(\frac{dr^2}{1 + g^2 r^2 (H_0H_1H_2) - g^2\alpha^2/r^4} + r^2 d\Omega^2\right)\,. \end{aligned} \tag{4.13}$$

The gauge fields are then given by

$$A^I_t = H_I^{-1}, \quad A^I_m = w_m A^I_t\,. \tag{4.14}$$

[5] We apologize for using the same symbol for one of the moduli and the function appearing in the metric, but the meaning should be clear from the context.

For de Sitter space, one can obtain multi-centered solutions where $e^{6U} = H_0 H_1 H_2$ in (4.5) with

$$H_0 = 3V_0 + \sum_{i=1}^{N} \frac{q_{0i}}{|\vec{x}_i - \vec{x}|^2} e^{-2gt} \,,$$
$$H_1 = 3V_1 + \sum_{i=1}^{N} \frac{q_{1i}}{|\vec{x}_i - \vec{x}|^2} e^{-2gt} \,,$$
$$H_2 = 3V_2 + \sum_{i=1}^{N} \frac{q_{2i}}{|\vec{x}_i - \vec{x}|^2} e^{-2gt} \,. \tag{4.15}$$

5. Physical discussion

5.1 Horizons, temperature, entropy and closed timelike curves

To discuss the conditions for the existence of horizons for the solution (4.9), we limit ourselves to the case without vector multiplets, where one has

$$e^{2U} = 1 + \frac{q}{r^2} \,. \tag{5.1}$$

From (4.9), we see that horizons occur whenever

$$1 + g^2 r^2 e^{6U} - \frac{g^2 \alpha^2}{r^4} = 0 \,, \tag{5.2}$$

which yields a third order equation for r^2. We define the dimensionless parameters

$$a \equiv \alpha g^3 \,, \qquad \rho = q g^2 \,, \tag{5.3}$$

as well as the critical rotation parameter

$$a_c^2 = \rho^2 + \frac{2}{3}\rho + \frac{2}{27} - \frac{2}{27}(1 + 6\rho)^{3/2} \,. \tag{5.4}$$

For $a^2 < a_c^2$ one finds no horizons, whereas for $\rho^2 \geq a^2 > a_c^2$ one gets an inner and an outer horizon. For $a^2 > \rho^2$ the inner horizon disappears, and we have only one horizon. For $a^2 = a_c^2$, the two horizons coalesce, and one is left with an extremal black hole with zero Hawking temperature.

As mentioned in the introduction, the solution (4.9) admits closed timelike curves outside the horizon. To see this, we note that the vector $\partial_\phi - \partial_\psi$, which has closed orbits, becomes timelike for

$$r^2 e^{6U} - \frac{\alpha^2}{r^4} < 0 \,. \tag{5.5}$$

One easily sees that this can occur outside the horizon. In particular, for the case without vector multiplets, (5.5) is satisfied for $r^2 < r_L^2 \equiv \alpha^{2/3} - q$, and one has $r_L > r_+$, where r_+ is the location of the horizon. This causality violation already appears in the ungauged case ($g = 0$), i.e. in the BMPV black hole [22], where

JHEP02(2001)031

closed timelike curves exist outside the horizon if $\alpha^2 > q^3$ ("over-rotating" black hole). Various aspects of this "time machine" have been studied in [23, 24]. Among other things, the authors of [23] showed that causal geodesics cannot penetrate the $r = r_+$ surface for the over-rotating solution (repulson-like behaviour), and so the exterior region is geodesically complete (with respect to causal geodesics). Furthermore, in [24], it was shown that upon lifting of the over-rotating BMPV black hole to a solution of type IIB string theory and passing to the universal covering space, the causal anomalies disappear. A detailed discussion of the issues related with the causality violation encountered for the solution (4.9) will be presented in [25]. We mention here briefly some of the results: Also in the gauged case, the exterior region is geodesically complete; no causal geodesics cross the $r = r_+$ surface. The five-dimensional solution for the $STU = 1$ model can be lifted to a solution of type IIB supergravity in ten dimensions using the Kaluza-Klein ansatz of [26]. One then obtains a rotating D3-brane wrapping S^3. The resulting ten-dimensional configuration still has closed timelike curves, so in our case no resolution of causal anomalies in higher dimensions appears.

Note that the naively computed temperature and Bekenstein-Hawking entropy are given by

$$\begin{aligned} T &= \frac{r_+^3}{4\pi\sqrt{e^{6U(r_+)}r_+^6 - \alpha^2}} \frac{d}{dr}\left[1 + g^2 r^2 e^{6U} - \frac{g^2\alpha^2}{r^4}\right]_{|r=r_+}, \\ S_{BH} &= \frac{A_{\text{hor}}}{4G} = \frac{\pi^2}{2G}\sqrt{e^{6U(r_+)}r_+^6 - \alpha^2}\,. \end{aligned} \tag{5.6}$$

These quantities are imaginary and hence become senseless. For the over-rotating ungauged case, it was argued in [23] that the effective entropy should be considered to be zero. The reason for this was the vanishing of the absorption cross section for a massless Klein-Gordon field, which is a measure of the horizon area. Also for the solution (4.13) of the $STU = 1$ model, the absorption probability for a Klein-Gordon field vanishes [25], indicating that the effective temperature and entropy are both zero.

The causality violation in the spacetime (4.9) should translate on the dual CFT side into loss of unitarity (relation between macroscopic causality in the AdS bulk and microscopic unitarity in the boundary CFT). For the rotating BMPV black hole, which corresponds to a D1-D5-Brinkmann wave system in type IIB string theory [24], one has a description in terms of an $\mathcal{N} = 4$ two-dimensional superconformal field theory. The causality bound $\alpha^2 = q^3$ then corresponds to the unitarity bound in this CFT [24]. Probably the appearance of closed timelike curves can be avoided in the nonsupersymmetric generalization of (4.9), because there would be one more parameter (the nonextremality parameter μ) available. It should then be possible to shift the region of closed timelike curves beyond the horizon by choosing μ sufficiently large. It would be interesting to determine the resulting unitarity bound for $D = 4$, $\mathcal{N} = 4$

SYM theory, which would be obtained by requiring the absence of closed timelike curves outside the horizon. This unitarity bound could then be compared to the one coming from the superconformal algebra of the corresponding CFT. The classification of unitary representations of superconformal algebras typically implies inequalities on the conformal weights and R-charges [27]. On passing from four-dimensional Minkowski space to $\mathbb{R} \times S^3$ (where our CFT lives, cf. below), eigenvalues of the generator of dilatations D translate into eigenvalues of the hamiltonian H. Thus the inequalities mentioned above yield lower bounds on the black hole mass M, and these lower bounds are given in terms of the R-charges. As M typically is proportional to the nonextremality parameter μ, it is clear that by turning on a sufficiently large μ, the unitarity bound can be satisfied. Clearly the nonextremal generalization of (4.9) is not yet known, but there should exist a general solution which contains both (4.9) and the nonextremal uncharged Kerr-AdS black holes found in [9].

5.2 Stress tensor of the dual CFT

In computing quantities like the Euclidean action, which yields the thermodynamical potential relevant to the various ensembles, one usually encounters infrared divergences, which are regularized by subtracting a suitably chosen background. Such a procedure, however, in general is not unique; in some cases the choice of reference background is ambiguous, e. g. for hyperbolic AdS black holes [28, 29]. Recently, in order to regularize such divergences, a different procedure has been proposed [30, 31, 29].[6] This technique was inspired by the AdS/CFT correspondence, and consists in adding suitable counterterms I_{ct} to the action. These counterterms are built up with curvature invariants of a boundary $\partial\mathcal{M}$ (which is sent to infinity after the integration), and thus obviously they do not alter the bulk equations of motion. This kind of procedure, which will also be used in the present paper, has the advantage of being free of ambiguities, which, on the contrary, are present in the traditional approach in some particular cases, like mentioned above.

In the following discussion, we only consider the case without vector multiplets, and leave the general case for a future publication. The Euclidean action for Einstein-Maxwell-AdS gravity has the form[7]

$$\begin{aligned} I &= I_{\rm bulk} + I_{\rm surf} + I_{\rm ct} \\ &= -\frac{1}{16\pi G}\int_{\mathcal{M}} d^5x\sqrt{g}\left[R + 12g^2 - \frac{1}{12}F_{\mu\nu}F^{\mu\nu} + \frac{1}{108\sqrt{g}}\epsilon^{\mu\nu\rho\sigma\lambda}F_{\mu\nu}F_{\rho\sigma}A_\lambda\right] - \\ &\quad -\frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^4x\,\sqrt{h}\,K + I_{ct}\,. \end{aligned} \tag{5.7}$$

The second term is the Gibbons-Hawking boundary term, where h_{ab} is the boundary metric, and K denotes the trace of the extrinsic curvature K_{ab}.

[6] Cf. also [32].

[7] This action is related to that of [16] by rescaling the gauge fields of [16] by $1/(2\sqrt{3})$.

The action (5.7) differs from the familiar one by the presence of the last term, which contains all surface counterterms needed to insure the convergence of the integrals. In five dimensions it reads [29]

$$I_{ct} = \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^4x\sqrt{h}\left[3g + \frac{\mathcal{R}}{4g}\right], \tag{5.8}$$

$\mathcal{R}$ denoting the Ricci scalar of the boundary. Going back to lorentzian signature, one can now construct a divergence free stress tensor given by

$$T^{ab} = \frac{2}{\sqrt{-h}}\frac{\delta I}{\delta h_{ab}} = \frac{1}{8\pi G}\left[K^{ab} - h^{ab}K - 3gh^{ab} + \frac{G^{ab}}{2g}\right], \tag{5.9}$$

where G^{ab} denotes the Einstein tensor built up from h_{ab}. If we choose $\partial\mathcal{M}$ to be a four-surface of fixed r, then the metric on the manifold upon which the dual CFT resides is defined by

$$\gamma_{ab} = \lim_{r\to\infty}\frac{1}{g^2r^2}h_{ab}\,. \tag{5.10}$$

In our case, this yields

$$\gamma_{ab}dx^a dx^b = -dt^2 + \frac{1}{g^2}d\Omega_3^2\,, \tag{5.11}$$

where $d\Omega_3^2$ is the standard metric on the unit S^3. We see that the dual CFT is defined on the (nonrotating) manifold $\mathbb{R}\times S^3$. The field theory's stress tensor $\hat{T}^{ab}$ is related to the one in (5.9) by the rescaling [33]

$$\sqrt{-\gamma}\gamma_{ab}\hat{T}^{bc} = \lim_{r\to\infty}\sqrt{-h}h_{ab}T^{bc}\,, \tag{5.12}$$

which amounts to multiplying all expressions for T_{ab} given below by g^2r^2 before taking the limit $r\to\infty$.

Before we proceed, a short comment is in order. In five dimensions, in general one encounters also logarithmic divergences in the computation of the Euclidean action. These divergences, which cannot be removed by adding local counterterms, are related to the Weyl anomaly of the dual CFT [30]. In our case, the divergent term reads [30]

$$I_{\log} = -\frac{\ln\epsilon}{64\pi Gg^3}\int_{\mathcal{M}} d^4x\sqrt{\gamma}\left[\mathcal{R}_{ab}\mathcal{R}^{ab} - \frac{1}{3}\mathcal{R}^2\right], \tag{5.13}$$

where ϵ denotes a bulk infrared cutoff. For the metric (5.11), one finds that this term vanishes, so there is no conformal anomaly in the dual field theory. In principle, an additional logarithmically divergent term can appear in the Euclidean action (5.7) due to the Maxwell gauge field A_μ [34]. However, for the black holes considered here, this term also vanishes.[8]

[8]In our case, $F_{\mu\nu}F^{\mu\nu}$ falls off as r^{-6} for $r\to\infty$, whereas a logarithmic divergence would come from a term of order r^{-4} [34].

We now come to the computation of the stress tensor (5.9). Using (4.9), one obtains

$$
\begin{aligned}
8\pi G T_{tt} &= \frac{3}{8gr^2}(8\rho+1) + \mathcal{O}(r^{-4})\,, \\
8\pi G T_{t\phi} &= \frac{2g\alpha}{r^2}\sin^2\theta + \mathcal{O}(r^{-4})\,, \\
8\pi G T_{t\psi} &= -\frac{2g\alpha}{r^2}\cos^2\theta + \mathcal{O}(r^{-4})\,, \\
8\pi G T_{\phi\phi} &= \frac{\sin^2\theta}{8g^3r^2}(8\rho+1) + \mathcal{O}(r^{-4})\,, \\
8\pi G T_{\psi\psi} &= \frac{\cos^2\theta}{8g^3r^2}(8\rho+1) + \mathcal{O}(r^{-4})\,, \\
8\pi G T_{\phi\psi} &= \mathcal{O}(r^{-4})\,, \\
8\pi G T_{\theta\theta} &= \frac{1}{8g^3r^2}(8\rho+1) + \mathcal{O}(r^{-4})\,,
\end{aligned} \tag{5.14}
$$

all other components vanishing. Introducing the unit time-like four vector $v = (1,0,0,0)$ as well as the null vectors $w_\pm = (1,0,\pm g/\sin\theta, 0)$, $z_\pm = (1,0,0,\pm g/\cos\theta)$, one can write the field theory's stress tensor in the form[9]

$$
\begin{aligned}
8\pi G\hat{T}^{ab} = &\ \frac{g}{8}(8\rho+1)[4v^a v^b + \gamma^{ab}] - g^4\alpha\sin\theta[w_+^a w_+^b - w_-^a w_-^b] + \\
&+ g^4\alpha\cos\theta[z_+^a z_+^b - z_-^a z_-^b]\,.
\end{aligned} \tag{5.15}
$$

One easily checks that this tensor is conserved and traceless.
The stress tensor (5.14) can also be used to compute conserved quantities like mass and angular momenta of the black hole. To do this, we indicate by u^μ the unit normal vector of a spacelike hyper-surface ${}^4\mathcal{S}_t$ at constant t, and by Σ the spacelike intersection ${}^4\mathcal{S}_t \cap \partial\mathcal{M}$ embedded in $\partial\mathcal{M}$ with induced metric σ_{ab}. Then, for any Killing vector field ξ^μ there is an associated conserved charge

$$
Q_\xi = \int_\Sigma d^3x\,\sqrt{\sigma}\,u^\mu T_{\mu\nu}\xi^\nu\,. \tag{5.16}
$$

In this way one gets for the black hole mass

$$
M = Q_{\partial_t} = \frac{3\pi}{32Gg^2}(8\rho+1)\,, \tag{5.17}
$$

and for the angular momenta

$$
\begin{aligned}
J_\phi &= Q_{\partial_\phi} = \frac{\alpha\pi}{4G}\,, \\
J_\psi &= Q_{\partial_\psi} = -\frac{\alpha\pi}{4G}\,.
\end{aligned} \tag{5.18}
$$

The angular momenta coincide with those of the ungauged case calculated in [21].

[9] A similar form has been obtained for five-dimensional uncharged Kerr-AdS black holes in [7].

6. De Sitter case

We turn now to the discussion of the de Sitter black holes (4.7). These solutions could be relevant to the proposed duality [35] between the large N limit of euclidean four-dimensional $U(N)$ super Yang-Mills theory and type IIB* string theory in de Sitter space. For pure Einstein-Maxwell gravity with cosmological constant, some of the physical properties of the de Sitter black holes (4.7) have been discussed in [20]. The new feature which arises is the presence of a cosmological horizon at $r = r_c > r_+$, where $r = r_+$ is the location of the black hole event horizon.[10] In this section we will generalize some of the results of [20] to the case of additional vector and scalar fields.

6.1 Smarr formula

To obtain a Smarr-type formula for the de Sitter solutions, from which a first law of black hole mechanics can be deduced, we proceed along the lines of [36], where the four-dimensional Kerr-Newman-de Sitter black hole was considered. We start from the Killing identity

$$\nabla_\mu \nabla_\nu K^\mu = R_{\nu\rho} K^\rho = \Big(F^2_{\nu\rho} - \frac{1}{6} g_{\nu\rho} F^2\Big) K^\rho + G_{IJ} \partial_\nu X^I \partial_\rho X^J K^\rho + \frac{2}{3} g^2 V(X) K_\nu \,, \quad (6.1)$$

where K^μ is a Killing vector, $\nabla_{(\nu} K_{\mu)} = 0$. Note that in the second step we used the Einstein equation of motion following from the action (2.1). Integrating (6.1) on a spacelike hypersurface Σ_t from the black hole horizon r_+ to the cosmological horizon r_c, using Gauss' law gives

$$\frac{1}{2} \int_{\partial \Sigma_t} \nabla_\mu K_\nu d\Sigma^{\mu\nu} = \frac{2}{3} g^2 \int_{\Sigma_t} V(X) K_\nu d\Sigma^\nu + \quad (6.2)$$

$$+ \int_{\Sigma_t} (F^2_{\nu\rho} - \frac{1}{6} g_{\nu\rho} F^2 + G_{IJ} \partial_\nu X^I \partial_\rho X^J) K^\rho d\Sigma^\nu \,, \quad (6.3)$$

where the boundary $\partial\Sigma_t$ consists of the intersection of Σ_t with the black-hole and the cosmological horizon,

$$\partial \Sigma_t = S^3(r_+) \cup S^3(r_c) \,. \quad (6.4)$$

In a first step, we apply (6.2) to the Killing vectors ∂_ϕ and ∂_ψ, which we denote by $\tilde{K}^i$, $i = \phi, \psi$. Using $\tilde{K}^i_\nu d\Sigma^\nu = 0$, we get

$$\frac{1}{16\pi G} \int_{S^3(r_+)} \nabla_\mu \tilde{K}^i_\nu d\Sigma^{\mu\nu} + \frac{1}{16\pi G} \int_{S^3(r_c)} \nabla_\mu \tilde{K}^i_\nu d\Sigma^{\mu\nu} = \frac{1}{G} \int_{\Sigma_t} T_{\nu\rho} \tilde{K}^{i\rho} d\Sigma^\nu \,, \quad (6.5)$$

[10] For the de Sitter solutions, one can show that at least in the case without vector multiplets or for the $STU = 1$ model, no closed timelike curves appear in the region between the event horizon and the cosmological horizon.

where

$$T_{\nu\rho} = \frac{1}{8\pi}\left(F^2_{\nu\rho} - \frac{1}{4}g_{\nu\rho}F^2 + G_{IJ}\partial_\nu X^I\partial_\rho X^J - \frac{1}{2}g_{\nu\rho}G_{IJ}\partial_\lambda X^I\partial^\lambda X^J\right) \tag{6.6}$$

is the stress-energy tensor of the vector fields and scalars. The right-hand side of (6.5) can be interpreted as the angular momentum of the matter between the two horizons. Thus the second term on the left-hand side of (6.5) can be regarded as being the total angular momentum, J^i_c, contained in the cosmological horizon, and the first term on the left-hand side as the negative of the angular momentum J^i_{BH} of the black hole (cf. also discussion in [36]).

In a second step, we apply (6.2) to the Killing vector $K = \partial_t$. This yields

$$\begin{aligned}
&\frac{3}{32\pi G}\int_{S^3(r_+)}\nabla_\mu K_\nu d\Sigma^{\mu\nu} + \frac{3}{32\pi G}\int_{S^3(r_c)}\nabla_\mu K_\nu d\Sigma^{\mu\nu} \\
&= \frac{3g^2}{24\pi G}\int_{\Sigma_t} V(X)K_\nu d\Sigma^\nu + \frac{3}{2G}\int_{\Sigma_t}\left(T_{\nu\rho} - \frac{1}{3}Tg_{\nu\rho}\right)K^\rho d\Sigma^\nu\,.
\end{aligned} \tag{6.7}$$

The right-hand side of equation (6.7) represents, respectively, the contribution of the scalar potential energy and the matter kinetic energy to the mass within the cosmological horizon. Therefore, the second term on the left-hand side is identified as the total mass M_c within the cosmological horizon, and the first term as the negative of the black hole mass M_{BH}. As in ref. [37], the latter can be rewritten by expressing $K = \partial_t$ in terms of the null generator

$$l = \partial_t + \Omega^\phi_H\partial_\phi + \Omega^\psi_H\partial_\psi = \partial_t + \Omega^i_H\tilde{K}^i \tag{6.8}$$

of the black hole event horizon. Here, the Ω^i_H denote the angular velocities of the horizon. Using the definition of the surface gravity κ_H, which, by the zeroth law, is constant on the horizon, one obtains

$$M_{BH} = \frac{3}{16\pi G}\kappa_H A_H + \frac{3}{2}\Omega^i_H J^i_{BH}\,. \tag{6.9}$$

One therefore gets the Smarr-type formula

$$\begin{aligned}
M_c = &\frac{3}{16\pi G}\kappa_H A_H + \frac{3}{2}\Omega^\phi_H J^\phi_{BH} + \frac{3}{2}\Omega^\psi_H J^\psi_{BH} + \frac{3g^2}{24\pi G}\int_{\Sigma_t}V(X)K_\nu d\Sigma^\nu + \\
&+\frac{3}{2G}\int_{\Sigma_t}\left(T_{\nu\rho} - \frac{1}{3}Tg_{\nu\rho}\right)K^\rho d\Sigma^\nu.
\end{aligned} \tag{6.10}$$

7. Final remarks

In this paper we derived a class of general charged black hole solutions to gauged $N = 2$, $D = 5$ supergravity coupled to an arbitrary number of abelian vector multiplets. These black holes preserve half of the supersymmetries. For imaginary coupling constant of the gravitini (de Sitter case), we obtained also multicentered charged rotating solutions. The AdS black holes have some interesting properties, e.g. we saw that there is a minimum amount of rotation required to avoid naked singularties, similar to the four-dimensional case [38]. However, closed timelike curves appear outside the horizon. This causality violation indicates loss of unitarity in the dual superconformal field theory. It would be interesting to know the nonsupersymmetric generalizations of the solutions found here, because a sufficiently large nonextremality parameter would presumably shift the region of closed timelike curves beyond the horizon. A detailed discussion of the issues related with the encountered causality violation will be presented in [25].

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JHEP02(2001)031